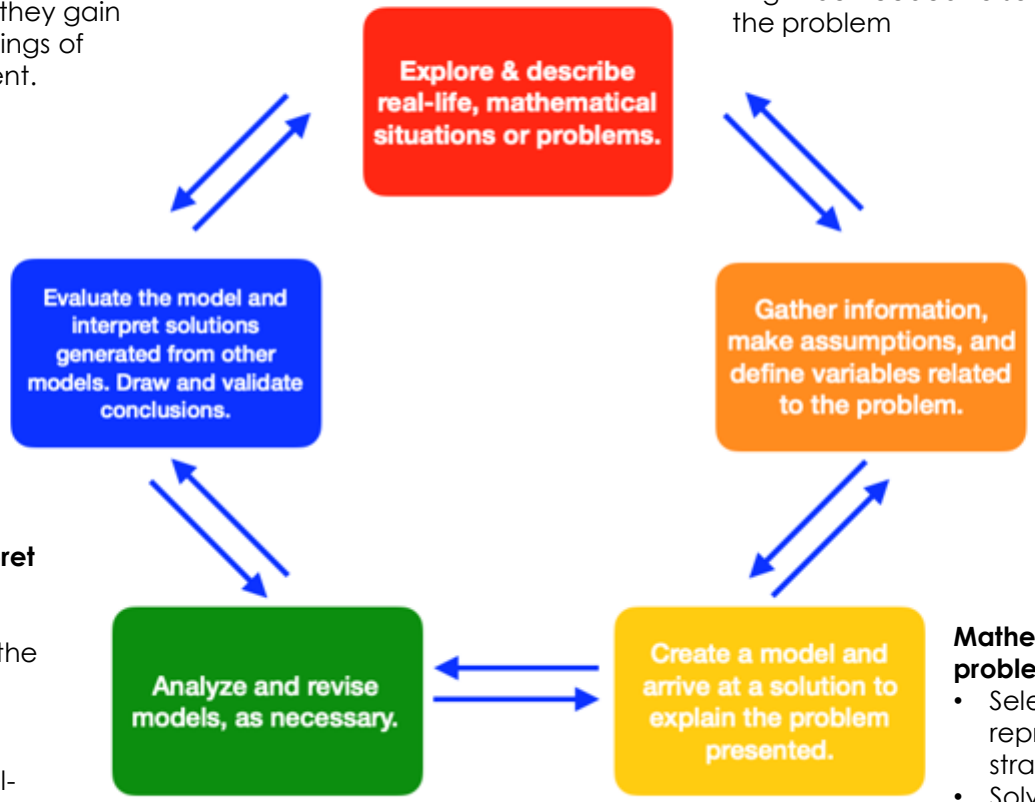
# Essential Instructional Guidance

The diagram below is a mathematical modeling framework depicting a cycle of how students can engage in mathematical modeling when solving a realistic problem or task.

## A Mathematical Modeling Framework

**Real World: More Questions?**
- More questions for exploration may arise as students reflect on the real-world solutions presented
- Students may choose to revisit and improve models as they gain deeper understandings of mathematics content.

**Real World: Students interpret the real-world problem**
- Make sense of the problem
- Determine what information might be needed to solve the problem




**Real/Mathematical World: Students reflect on models presented**
- Students share models and solutions to the real-world problem, explaining any assumptions made and information/variables considered for their model.
- Reflect on the real-world solutions
- Analyze and improve models

**Mathematical World: Students begin to use the language, tools, and structures of mathematics to solve the real-world problem.**
- Gather information
- Make assumptions
- Define variables

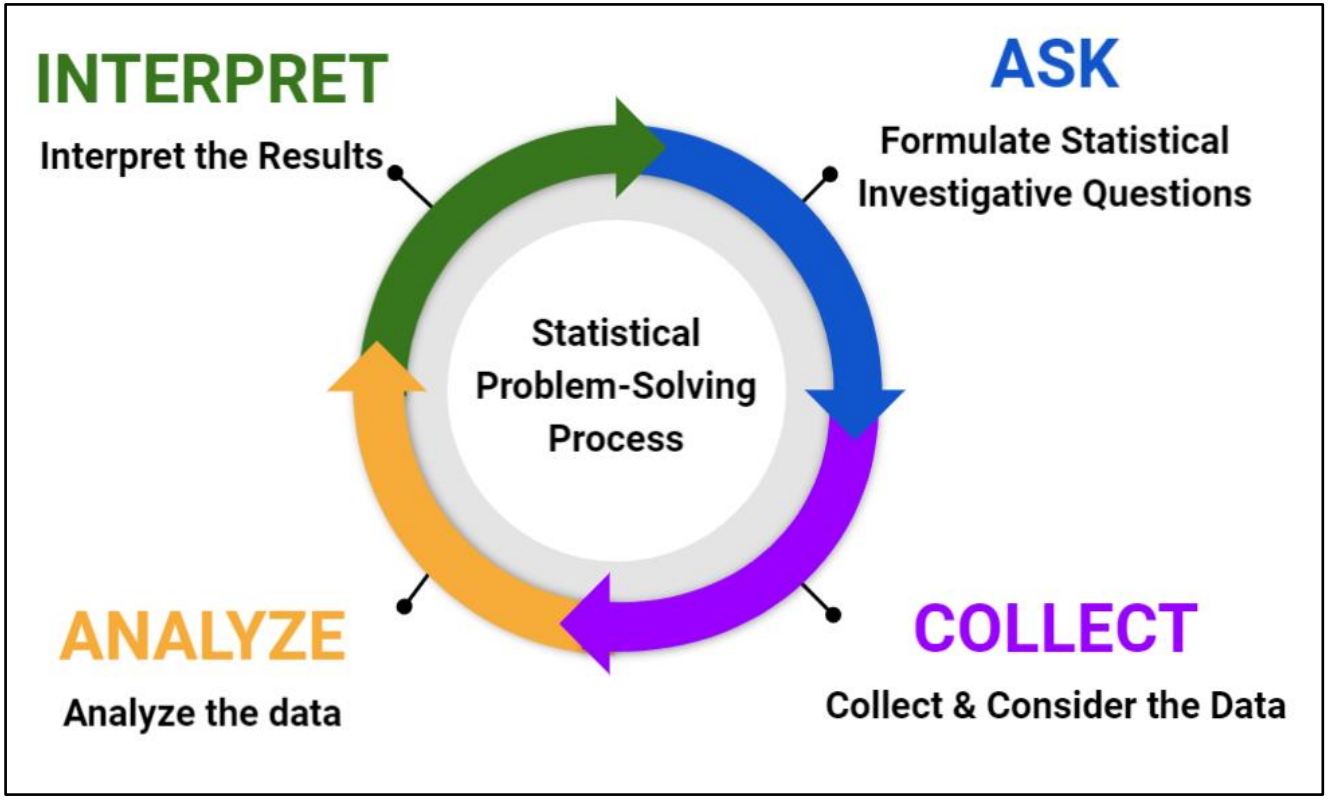
**Mathematical/Real World: Students interpret mathematical solution to the real-world context.**
- Interpret the mathematical solution in the context of the real-world problem
- Present the solution of the real-world problem
- As students analyze solutions in the real-world context, they may choose to revise models, and move back and forth between mathematical world and the real-world context

**Mathematical World: Students solve the problem using mathematics.**
- Select & use appropriate representations and mathematical strategies & methods
- Solve the problem and obtain a mathematical solution

# Essential Instructional Guidance

The **Framework for Statistical Reasoning** should be used in all grade levels and courses to guide learners through the sense-making process, ultimately leading to the goal of statistical literacy in all grade levels and courses. Reasoning with statistics provides a context that necessitates the learning and application of a variety of mathematical concepts.

The following **four-step statistical problem-solving process** can be used throughout each grade level and course to help learners develop a solid foundation in statistical reasoning and literacy:




I. **Formulate Statistical Investigative Questions**
   Ask questions that anticipate variability.
II. **Collect & Consider the Data**
   Ensure that data collection designs acknowledge variability.
III. **Analyze the Data**
   Make sense of data and communicate what the data mean using pictures (graphs) and words. Give an accounting of variability, as appropriate.
IV. **Interpret the Results**
   Answer statistical investigative questions based on the collected data.

# Essential Instructional Guidance

## Effective Mathematics Teaching Practices (Teacher Practices)

A strong math program involves engaging teaching that fosters meaningful learning through individual and group experiences, enhancing students' ability to grasp and apply mathematical concepts. Teaching math is intricate, demanding teachers to deeply comprehend the content and its grade-wise progression. Effective instructional strategies are vital for advancing math learning for all students. The following eight Mathematics Teaching Practices encapsulate essential skills drawn from research-based principles and emerging math teaching knowledge (NCTM, 2014).

| | | | |
|---|---|---|---|
| 1<br>**Establish mathematics goals to focus learning.** | 2<br>**Implement tasks that promote reasoning and problem solving.** | 3<br>**Use and connect mathematical representations.** | 4<br>**Facilitate meaningful mathematical discourse.** |
| 5<br>**Pose purposeful questions.** | 6<br>**Build procedural fluency from conceptual understanding.** | 7<br>**Support productive efforts of learning in mathematics.** | 8<br>**Elicit and use evidence of student thinking.** |

# Essential Instructional Guidance

## Effective Mathematics Teaching Practices

| Establish mathematics goals to focus learning. | Implement tasks that promote reasoning and problem solving. | Use and connect mathematical representations. | Facilitate meaningful mathematical discourse. |
|---|---|---|---|
| **Teachers should:**<br>• Consider broader goals, as well as the goals of the actual lesson, including the following:<br>  ○ What is to be learned?<br>  ○ Why is the goal important?<br>  ○ Where are students coming from? Where do students need to go?<br>  ○ How can learning be extended? | **Teachers should:**<br>Choose task that:<br>• are built on current student understandings.<br>• have various entry points with multiple ways for the problems to be solved.<br>• are interesting to students (e.g., evolve from students' thinking; connect to real world mathematics) | **Teachers should:**<br>• Use tasks that allow students to use a variety of representations.<br>• Encourage the use of different representations, including concrete models, pictures, words, and numbers, that support students in explaining their thinking and reasoning.<br>• Facilitate flexibility in student reasoning. | **Teachers should:**<br>• Engage students in explaining their mathematical reasoning in small groups and classroom situations.<br>• Facilitate discussion among students that support making sense of a variety of strategies and approaches.<br>• Scaffold classroom discussions so that connections between representations and mathematical ideas take place. |
| **Students should:**<br>• Make sense of new concepts and skills; including connections to concepts/big ideas learned in previous grades.<br>• Experience connections among the strands, overall and specific expectations.<br>• Deepen their understanding and expect mathematics to make sense. | **Students should:**<br>• Work to make sense out of the task and persevere in solving problems.<br>• Use a variety of models and materials to make sense of the mathematics in the task.<br>• Convince themselves and others the answer is reasonable. | **Students should:**<br>• Use materials to make sense out of problem situations.<br>• Connect representations to mathematical ideas and structures of big ideas, including operational sense with whole numbers, fractions and decimals.<br>• Use multiple representations to determine strategies that work best for them. | **Students should:**<br>• Explain the ideas and reasoning in small groups and with the entire class.<br>• Listen to the reasoning of others.<br>• Ask questions of others to make sense of their ideas. |

Adapted from John Hattie's (2017, p. 244) summation from Principles to Action (National Council of Teachers of Mathematics, 2014)

# Essential Instructional Guidance

## Effective Mathematics Teaching Practices

| Pose purposeful questions. | Build procedural fluency from conceptual understanding. | Support productive efforts of learning in mathematics. | Elicit and use evidence of student thinking. |
|---|---|---|---|
| **Teachers should:**<br>• Ask questions that build on and extend student thinking.<br>• Facilitate discussion among students that support making sense of a variety of strategies and approaches.<br>• Scaffold classroom discussions so that connections between representations and mathematical ideas take place. | **Teachers should:**<br>• Provide opportunities for students to reason about mathematical ideas.<br>• Expect students to explain why their strategies work.<br>• Connect student methods to efficient procedures as appropriate. | **Teachers should:**<br>• Support students in building their understanding without showing and telling a procedure but rather focusing on the important mathematical ideas.<br>• Ask questions that scaffold and advance student thinking.<br>• Build questions and plans lessons based on important student misconceptions rather than focusing on the correct answer.<br>• Recognize the importance of effort as students work to make sense of new ideas. | **Teachers should:**<br>• Determine what to look for in gathering evidence of student learning.<br>• Pose questions and answers student questions that provide information about student understanding and reasoning.<br>• Uses evidence to determine next steps of instruction. |
| **Students should:**<br>• Think more deeply about the process of the mathematics rather than simply focusing on the answer.<br>• Listen to and comment on the explanations of others in the class. | **Students should:**<br>• Understand and explain the procedures they are using and why they work.<br>• Use a variety of strategies to solve problems and make sense of the mathematical tasks.<br>• Do not rely on shortcuts or tricks to do mathematics. | **Students should:**<br>• Stick to tasks and recognize that effortful learning is part of making sense.<br>• Ask questions that will help to better understand the task.<br>• Support each other with ideas rather than telling others the answer or how to solve the problem. | **Students should:**<br>• Accept reasoning and understanding are as important as the answer to the problem.<br>• Use mistakes and misconceptions to rethink understanding.<br>• Ask questions to clarify confusion or misunderstanding.<br>• Assess progress toward developing mathematical understanding. |

Adapted from John Hattie's (2017, p. 244) summation from Principles to Action (National Council of Teachers of Mathematics, 2014)

# Essential Instructional Guidance

## Mathematical Practices (Student Habits of Mind)

The Mathematical Practices describe the reasoning behaviors students should develop as they build an understanding of mathematics – the "habits of mind" that help students become mathematical thinkers. There are eight standards, which apply to all grade levels and conceptual categories.

These mathematical practices describe how students should engage with the mathematics content for their grade level. Developing these habits of mind builds students' capacity to become mathematical thinkers. These practices can be applied individually or together in mathematics lessons, and no particular order is required. In well-designed lessons, there are often two or more Mathematical Practices present.

**MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.**

| | | | |
|---|---|---|---|
| **1** Make sense of problems and persevere in solving them. | **2** Reason abstractly and quantitatively | **3** Construct viable arguments and critique the reasoning of others. | **4** Model with mathematics. |
| **5** Use appropriate tools strategically. | **6** Attend to precision. | **7** Look for and make use of structure. | **8** Look for and express regularity in repeated reasoning. |

# Essential Instructional Guidance

## Mathematical Practices

| Make sense of problems and persevere in solving them. | Reason abstractly and quantitatively. | Construct viable arguments and critique the reasoning of others. | Model with mathematics. |
|---|---|---|---|
| **Students are:**<br>• Working and reading rich problems carefully<br>• Drawing pictures, diagrams, tables, or using objects to make sense of the problem<br>• Discussing the meaning of the problem with classmates<br>• Making choices about which solution path to take<br>• Trying out potential solution paths and making changes as needed<br>• Checking answers and making sure solutions are reasonable and make sense<br>• Exploring other ways to solve the problem<br>• Persisting in efforts to solve challenging problems, even after reaching a point of frustration. | **Students are:**<br>• Using mathematical symbols to represent situations<br>• Taking quantities out of context to work with them (decontextualizing)<br>• Putting quantities back in context to see if they make sense (contextualizing)<br>• Considering units when determining if the answer makes sense in terms of the situation | **Students are:**<br>• Making and testing conjectures<br>• Explaining and justifying their thinking using words, objects, and drawings<br>• Listening to the ideas of others and deciding if they make sense<br>• Asking useful questions<br>• Identifying flaws in logic when responding to the arguments of others<br>• Elaborating with a second sentence (spontaneously or prompted by the teacher or another student) to explain their thinking and connect it to their first sentence.<br>• Talking about and asking questions about each other's thinking, in order to clarify or improve their own mathematical understanding.<br>• Revising their work based upon the justification and explanations of others. | **Students are:**<br>• Using mathematical models (i.e., formulas, equations, symbols) to solve problems in the world<br>• Using appropriate tools such as objects, drawings, and tables to create mathematical models<br>• Making connections between different mathematical representations (concrete, verbal, algebraic, numerical, graphical, pictorial, etc.)<br>• Checking to see if an answer makes sense within the context of a situation and changing the model as needed |

# Essential Instructional Guidance

## Mathematical Practices

| Use appropriate tools strategically. | Attend to precision. | Look for and make use of structure. | Look for and express regularity in repeated reasoning. |
|---|---|---|---|
| **Students are:**<br>• Using technological tools to explore and deepen understanding of concepts<br>• Deciding which tool will best help solve the problem. Examples may include:<br>o Calculator<br>o Concrete models<br>o Digital Technology<br>o Pencil and paper<br>o Ruler, compass, protractor<br>• Estimating solutions before using a tool<br>• Comparing estimates to solutions to see if the tool was effective | **Students are:**<br>• Communicating precisely using clear language and accurate mathematics vocabulary<br>• Deciding when to estimate or give an exact answer<br>• Calculating accurately and efficiently, expressing answers with an appropriate degree of precision<br>• Using appropriate units; appropriately labeling diagrams and graphs | **Students are:**<br>• Finding structure and patterns in numbers<br>• Finding structure and patterns in diagrams and graphs<br>• Using patterns to make rules about math<br>• Using these math rules to help them solve problems | **Students are:**<br>• Looking for patterns when working with numbers, diagrams, tables, and graphs<br>• Observing when calculations are repeated<br>• Using observations from repeated calculations to take shortcuts<br>• Using repeated reasoning to develop flexible thinking strategies |

**Locate additional information at the following link:**
https://www.georgiastandards.org/Georgia-Standards/Documents/Standards-for-Mathematical-Practice-Look-Fors.pdf

# Essential Instructional Guidance

## Effective Instructional Practices Resource

These resources provide mathematics leaders and teachers with a quick reference of highlighted **instructional practices** along with characteristics of teacher and student behaviors when the practice is implemented.

| Evidence-Based Practices | | |
|---|---|---|
| Three-Act Task | Cognitively Guided Instruction | High-Leverage Practices |
| Incorporating the 8 Mathematical Practices | Modeling with Mathematics | Number Talks |
| Numberless Word Problems | Numeracy Intervention Resources | Patient Problem Solving |
| Pattern Talks | Positive Mathematics Mindsets and Productive Efforts | Problem-Based Learning |
| Standards-Based Grading and Intervention Models | Statistical Reasoning and Data Literacy | Which One Doesn't Belong |

https://lor2.gadoe.org/gadoe/file/e7967472-6a87-4062-a788-0715d40ddcab/1/Mathematics-Effective-Instructional-Practices-Resource-for-Leaders.pdf